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MTH101 Calculus and Analytical Geometry

Final Term Examination – Spring 2005

Time Allowed: 150 Minutes

Maximum Time Allowed: (150 mins)

Please read the following instructions carefully before attempting any of the questions:

- 1. Pasting the equations of math type from word file into software may cause some visibility problem, so please note that do not copy equations of math type into software from word file. Paste the equations from math type directly into software.**
- 2. Do not ask any questions about the contents of this examination from anyone.**
 - a. If you think that there is something wrong with any of the questions, attempt it to the best of your understanding.**
 - b. If you believe that some essential piece of information is missing, make an appropriate assumption and use it to solve the problem.**
 - c. Write all steps, missing steps may lead to deduction of marks.**

****WARNING: Please note that Virtual University takes serious note of unfair means. Anyone found involved in cheating will get an `F` grade in this course.**

Total Marks: 65

Total Questions: 12

Question No. 1

Marks : 05

Using the concept of infinite series expresses 4.212121..... as a ratio of integers.

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 2

Marks : 03

If

$$y = f(x)$$

then

$$f'_x(x_0)$$

is the function whose value at x_0 is the average rate of change of y with respect to x at point x_0 .

Question No. 3

Marks : 10

Determine whether the series converges or diverges. If it converges, find the sum

$$\sum_{k=1}^{\infty} \left(-\frac{3}{4}\right)^{k-1}$$

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 4

Marks : 07

Find the area of the region enclosed by :

$$y = x^3 + 5x^2 + 3, \quad y = x^2 + 7x + 3,$$

$$x = 0, \quad x = 3.$$

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 5

Marks : 05

Find the slope of the curve $y = (2x^2 + 7x - 2)^2$ at the point $x = -2$.

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 6

Marks : 03

The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is

- ☐ Converges
- ☐ Absolutely converges
- ☐ Diverges
- ☐ None of the above

Question No. 7

Marks : 03

If the $\lim (f(x) / g(x))$ is not an indeterminate form then L ' Hopital's rule cannot be applied

- ☐ True
- ☐ False

Question No. 8

Marks : 03

Two non vertical lines with slopes m_1 and m_2 respectively are parallel if and only if

- ☐ $m_1 m_2 = 1$
- ☐ $m_1 m_2 = -1$
- ☐ $\frac{m_1}{m_2} = 1$
- ☐ $\frac{m_1}{m_2} = -1$
- ☐

Question No. 9

Marks : 08

Solve the following integral

$$\int \frac{z}{\sqrt[3]{z^2 + 1}} dz$$

by substitution method.

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 10

Marks : 05

Find the limit using L' Hopital's Rule

$$\lim_{x \rightarrow \pi^+} \frac{\sin x}{x - \pi}$$

Note: In order to get the maximum marks you have to show all the necessary steps

Question No. 11

Marks : 10

$$f(x) = \begin{cases} x^2 + 1, & 0 < x \leq 1 \\ x, & 1 < x \leq 4 \\ 2x + 1, & 4 \leq x \end{cases}$$

Is $f(x)$ continuous at $x = 1$?

Write all necessary steps and justify your answer.

Note: In order to get the maximum marks you have to show all the necessary steps.

Question No. 12

Marks : 03

Which of the following statement is true

Let f be a function that is defined at all points in the interval $[a, b]$.

- ☐ If f is continuous on $[a, b]$, then f is integrable on $[a, b]$
 - ☐ If f is bounded on $[a, b]$ and has only finitely many points of discontinuity on $[a, b]$, then f is integrable on $[a, b]$
 - ☐ If f is not bounded on $[a, b]$, then f is not integrable on $[a, b]$.
 - ☐ All of above are true
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